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Existence and global attractivity of positive periodic solutions of functional differential equations with feedback control

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Abstract

Sufficient conditions are obtained for the existence and global attractivity of positive periodic solutions of the delay differential system with feedback control

$$
\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= -b(t)x(t) + F(t, x(t-\tau_1(t)), \ldots, x(t-\tau_n(t)), u(t-\delta(t))),\\
\frac{\mathrm{d}u}{\mathrm{d}t} &= -\eta(t)u(t) + a(t)x(t-\sigma(t)).
\end{aligned}
$$

The method involves the application of Krasnoselskii's fixed point theorem and estimates of uniform upper and lower bounds of solutions. When these results are applied to some special delay population models with multiple delays, some new results are obtained and some known results are generalized.

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Keywords: Delay differential equations; Feedback control; Positive periodic solutions; Global attractivity

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1. Introduction

If the environment is not temporally constant (e.g., seasonal effects of weather, food supplies, mating habits, etc.), then the parameters become time dependent. It has been suggested by Nicholson [24] that any periodic change of climate tends to impose its period upon oscillations of internal origin or to cause such oscillations to have a harmonic relation to periodic climatic changes. Pianka [26] discussed the relevance of periodic environment to evolutionary theory.

Nisbet and Gurney [25] considered a periodic delay logistic equation and carried out a numerical study of the influence of the periodicity in the environment on the intrinsic oscillations of the equation such as those caused by the time delay. Rosen [28] noted the existence of a relation between the period of the periodic carrying capacity and the delay of the logistic equation. Zhang and Gopalsamy [38] assumed that the intrinsic growth rate and the carrying capacity are periodic functions of a period ω and that the delay is an integer multiple of the period of the environment. Following the techniques of Zhang and Gopalsamy [38], people have studied some well-known delay models with periodic coefficients and delay, such as the periodic Nicholson's blowflies model [30], the periodic Allee effect model [18,36], the periodic food-limited model [5,6], the periodic Wazewska-Czyzewska and Lasota model [9,10,27,34], etc. In all these papers, the delays are assumed to be integral multiples of periods of the environment.

Ecosystems in the real world are continuously distributed by unpredictable forces which can result in changes in the biological parameters such as survival rates. Of practical interest in ecology is the question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables. The control variables discussed in most literatures are constants or time dependent (see [19,20]). In 1993, Gopalsamy and Weng [7] introduced a feedback control variable into the delayed logistic model and discussed the asymptotic behavior of solutions in logistic models with feedback controls, in which the control variables satisfy certain differential equation. We also refer to Lalli et al. [17], Liao [22], Tang and Zou [35], Yang and Jiang [37] for further study on delay equations with feedback control.

Recently, Huo and Li [12] considered the following general nonlinear nonautonomous delay differential system with feedback control

$$
\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= F(t, x(t-\tau_1(t)), \ldots, x(t-\tau_n(t)), u(t-\delta(t))),\\
\frac{\mathrm{d}u}{\mathrm{d}t} &= -\eta(t)u(t) + a(t)x(t-\sigma(t)),
\end{aligned}
\tag{1.1}
$$

which contains many bio-mathematical models of delay differential equations, such as logistic model with several delays and feedback control

$$
\begin{aligned}
\frac{\mathrm{d}x(t)}{\mathrm{d}t} &= x(t)\left[r(t) - \sum_{i=1}^{n} a_i(t)x(t-\tau_i(t)) - c(t)u(t-\delta(t))\right],\\
\frac{\mathrm{d}u(t)}{\mathrm{d}t} &= -\eta(t)u(t) + a(t)x(t-\sigma(t)),
\end{aligned}
\tag{1.2}
$$

the multiplicative delay logistic model with feedback control

$$\begin{aligned}\frac{\mathrm{d}x(t)}{\mathrm{d}t} &= r(t)x(t)\left[1-\prod_{i=1}^{n}\frac{x(t-\tau_i(t))}{K(t)}-c(t)u(t-\delta(t))\right],\\ \frac{\mathrm{d}u(t)}{\mathrm{d}t} &= -\eta(t)u(t)+a(t)x(t-\sigma(t)),\end{aligned}\tag{1.3}$$

the so-called Michaelis–Menton single-species growth model with feedback control

$$\begin{aligned}\frac{\mathrm{d}x(t)}{\mathrm{d}t} &= r(t)x(t)\left[1-\sum_{i=1}^{n}\frac{a_i(t)x(t-\tau_i(t))}{1+c_i(t)x(t-\tau_i(t))}-c(t)u(t-\delta(t))\right],\\ \frac{\mathrm{d}u(t)}{\mathrm{d}t} &= -\eta(t)u(t)+a(t)x(t-\sigma(t)),\end{aligned}\tag{1.4}$$

The coincidence degree theory [3] has been used to establish the existence of periodic solutions in (1.1). Stability of periodic solutions of (1.1) was also obtained when the delays are constants.

However, Eq. (1.1) does not contain the periodic delay Hematopoiosis model [23,29,31]

$$\begin{aligned}\frac{\mathrm{d}x(t)}{\mathrm{d}t} &= -b(t)x(t)+\frac{\beta(t)}{1+[x(t-\tau(t))]^m}+c(t)u(t-\delta(t)),\\ \frac{\mathrm{d}u}{\mathrm{d}t} &= -\eta(t)u(t)+a(t)x(t-\sigma(t)),\end{aligned}\tag{1.5}$$

the periodic survival red blood cells model [32,33]

$$\begin{aligned}\frac{\mathrm{d}x(t)}{\mathrm{d}t} &= -b(t)x(t)+\beta(t)\exp(-x(t-\tau(t)))+c(t)u(t-\delta(t)),\\ \frac{\mathrm{d}u}{\mathrm{d}t} &= -\eta(t)u(t)+a(t)x(t-\sigma(t))\end{aligned}\tag{1.6}$$

and the delay nonlinear equation [13]

$$\begin{aligned}\dot{x}(t) &= a(t)x(t)+\int_{-\infty}^{+\infty}C(s)g(t,x(t-\tau_0(t)),x(t-\tau_1(t)),x(t+s))\,\mathrm{d}s+c(t)u(t-\delta(t)),\\ u'(t) &= -\eta(t)u(t)+b(t)x(t-\delta(t)).\end{aligned}\tag{1.7}$$

Motivated by the above question, in this paper, we consider the following general nonlinear nonautonomous delay differential system with feedback control

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= -b(t)x(t)+F(t,x(t-\tau_1(t)),\ldots,x(t-\tau_n(t)),u(t-\delta(t))),\\ \frac{\mathrm{d}u}{\mathrm{d}t} &= -\eta(t)u(t)+a(t)x(t-\sigma(t))\end{aligned}\tag{1.8}$$

and

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= b(t)x(t)-F(t,x(t-\tau_1(t)),\ldots,x(t-\tau_n(t)),u(t-\delta(t))),\\ \frac{\mathrm{d}u}{\mathrm{d}t} &= -\eta(t)u(t)+a(t)x(t-\sigma(t)),\end{aligned}\tag{1.9}$$

where $F(t, z_1, z_2, \ldots, z_n, z_{n+1}) \in C(R^{n+2}, [0, \infty))$, $\tau_i(t)$ $(i = 1, 2, \ldots, n)$, $\delta(t)$, $\sigma(t) \in C(R, R)$, $\eta(t)$, $a(t)$, $b(t) \in C(R, (0, \infty))$, all functions are ω-periodic in t and $\omega > 0$ is a constant. By employing the Krasnoselskii's fixed point theorem and constructing some suitable Liapunov functionals, we obtain sufficient conditions of the existence and global attractivity of periodic solutions for system (1.8). Then we apply the obtained results to some delayed population models with multiple delays and feedback control, such as the periodic logistic equation with several delays [4,8,11,15] and the periodic delay Hematopoiosis model [23,29,31]. Some new results are obtained and some known results are generalized.

2. Main results

In this section, we establish the existence and global attractivity of positive periodic solutions of Eq. (1.8) by applying Krasnoselskii's fixed point theorem (see [2,14]) on cones. A compact operator will be constructed. It will be shown that the operator has a fixed point, which corresponds to a periodic solution of (1.8).

Definition 2.1. Let X be a Banach space and K be a closed, nonempty subset of X. K is a cone if

(i) $\alpha u + \beta v \in K$ for all $u, v \in K$ and all $\alpha, \beta \geqslant 0$ and
(ii) $u, -u \in K$ imply $u = 0$.

Lemma 2.1 (*Deimling [2] and Krasnoselskii [14]*). *Let X be a Banach space, and let $K \subset X$ be a cone in X. Assume that Ω_1, Ω_2 are open subsets of X with $0 \in \Omega_1$, $\bar{\Omega}_1 \subset \Omega_2$, and let*

$$\Phi : K \cap (\bar{\Omega}_2 \backslash \Omega_1) \to K$$

be a completely continuous operator such that either

(i) $\|\Phi y\| \leqslant \|y\|$ $\forall y \in K \cap \partial\Omega_1$ *and* $\|\Phi y\| \geqslant \|y\|$ $\forall y \in K \cap \partial\Omega_2$; *or*
(ii) $\|\Phi y\| \geqslant \|y\|$ $\forall y \in K \cap \partial\Omega_1$ *and* $\|\Phi y\| \leqslant \|y\|$ $\forall y \in K \cap \partial\Omega_2$.

Then Φ has a fixed point in $K \cap (\bar{\Omega}_2 \backslash \Omega_1)$.

Now we state our fundamental theorem about the existence of positive ω-periodic solutions of system (1.8).

Theorem 2.1. *Suppose that*

$$\lim_{|v| \to 0} \max_{t \in [0, \omega]} \frac{F(t, v)}{|v|} = 0, \qquad \lim_{|v| \to \infty, v_j \geqslant \sigma |v|, 1 \leqslant j \leqslant n+1} \min_{t \in [0, \omega]} \frac{F(t, v)}{|v|} = \infty \tag{2.1}$$

or

$$\lim_{|v| \to 0, v_j \geqslant \sigma |v|, 1 \leqslant j \leqslant n+1} \min_{t \in [0, \omega]} \frac{F(t, v)}{|v|} = \infty, \qquad \lim_{|v| \to \infty} \max_{t \in [0, \omega]} \frac{F(t, v)}{|v|} = 0, \tag{2.2}$$

where

$$\sigma := \exp\left(-2 \int_0^\omega b(s)\, \mathrm{d}s\right),$$

$v=(v_1,\ldots,v_{n+1})\in[0,\infty)^{n+1}$ *and* $|v|=\max\{v_1,\ldots,v_{n+1}\}$. *Then system* (1.8) *has at least one positive* ω*-periodic solution.*

Proof. Since each ω-periodic solution of the equation

$$\frac{\mathrm{d}u(t)}{\mathrm{d}t}=-\eta(t)u(t)+a(t)x(t-\sigma(t))$$

is equivalent to that of the equation

$$u(t)=\int_t^{t+\omega} g(t,s)a(s)x(s-\sigma(s))\,\mathrm{d}s := (\Phi x)(t) \tag{2.3}$$

and vice versa, where

$$g(t,s)=\frac{\exp(\int_t^s \eta(r)\,\mathrm{d}r)}{\exp(\int_0^\omega \eta(r)\,\mathrm{d}r)-1}.$$

It is easy to see that $u(t+\omega)=u(t)$. In fact,

$$\begin{aligned}u(t+\omega)&=\int_{t+\omega}^{t+2\omega} g(t+\omega,s)a(s)x(s-\sigma(s))\,\mathrm{d}s\\&=\int_t^{t+\omega} g(t+\omega,\xi+\omega)a(\xi+\omega)x(\xi+\omega-\sigma(\xi+\omega))\,\mathrm{d}\xi\\&=\int_t^{t+\omega} g(t+\omega,\xi+\omega)a(\xi)x(\xi-\sigma(\xi))\,\mathrm{d}\xi,\end{aligned}$$

but

$$\begin{aligned}g(t+\omega,\xi+\omega)&=\frac{\exp(\int_{t+\omega}^{\xi+\omega}\eta(r)\,\mathrm{d}r)}{\exp(\int_0^\omega \eta(r)\,\mathrm{d}r)-1}=\frac{\exp(\int_t^{\xi}\eta(\zeta+\omega)\,\mathrm{d}\zeta)}{\exp(\int_0^\omega \eta(r)\,\mathrm{d}r)-1}\\&=\frac{\exp(\int_t^{\xi}\eta(\zeta)\,\mathrm{d}\zeta)}{\exp(\int_0^\omega \eta(r)\,\mathrm{d}r)-1}=g(t,\xi).\end{aligned}$$

Hence,

$$\begin{aligned}u(t+\omega)&=\int_t^{t+\omega} g(t+\omega,\xi+\omega)a(\xi)x(\xi-\sigma(\xi))\,\mathrm{d}\xi\\&=\int_t^{t+\omega} g(t,\xi)a(\xi)x(\xi-\sigma(\xi))\,\mathrm{d}\xi=u(t).\end{aligned}$$

Therefore, the existence of ω-periodic solution of (1.8) is equivalent to that of the equation

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}=-b(t)x(t)+F(t,x(t-\tau_1(t)),\ldots,x(t-\tau_n(t)),(\Phi x)(t-\delta(t))). \tag{2.4}$$

For $(t,s)\in\mathbb{R}^2$, define

$$G(t,s)=\frac{\exp(-\int_s^t b(v)\,\mathrm{d}v)}{\exp(\int_0^\omega b(v)\,\mathrm{d}v)-1}. \tag{2.5}$$

It is clear that $G(t,s)>0$ and $G(t,s)=G(t+\omega,s+\omega)$ for all $(t,s)\in\mathbb{R}^2$.

We introduce two sets with ω and σ as given above

$$\begin{aligned}
&C_\omega=\{x\in C(\mathbb{R},\mathbb{R}^n):\ x(t+\omega)=x(t),\ t\in\mathbb{R}\},\\
&K=\{x\in C_\omega : x(t)\geqslant\sigma\|x\|,\ t\in[0,\omega]\}.
\end{aligned}\tag{2.6}$$

One may readily verify that K is a cone.

Now we define an operator $\Psi: K\to K$ as

$$(\Psi x)(t)=\int_t^{t+\omega} G(t,s)F(s,x(s-\tau_1(s)),\ldots,x(s-\tau_n(s)),(\Phi x)(s-\delta(s)))\,\mathrm{d}s$$

for $x\in K$, $t\in\mathbb{R}$, where $G(t,s)$ is defined following (2.5).

Claim 1. *$\Psi: K\to K$ is well-defined.*

In fact, for each $x\in K$, since $F(t,v)$ is a continuous function of t, we have $(\Psi x)(t)$ is continuous in t and

$$\begin{aligned}
(\Psi x)(t+\omega)&=\int_{t+\omega}^{t+2\omega} G(t+\omega,s)F(s,x(s-\tau_1(s)),\ldots,x(s-\tau_n(s)),(\Phi x)(s-\delta(s)))\,\mathrm{d}s\\
&=\int_t^{t+\omega} G(t+\omega,v+\omega)F(v+\omega,x(v+\omega-\tau_1(v)),\ldots,x(v+\omega-\tau_n(v)),(\Phi x)\\
&\quad\times(v+\omega-\delta(v)))\,\mathrm{d}v\\
&=\int_t^{t+\omega} G(t,v)F(v,x(v-\tau_1(v)),\ldots,x(v-\tau_n(v)),(\Phi x)(v-\delta(v)))\,\mathrm{d}v\\
&=(\Psi x)(t).
\end{aligned}$$

Thus, $(\Psi x)\in C_\omega$. Observe that

$$p:=\frac{\exp(-\int_0^\omega b(v)\,\mathrm{d}v)}{\exp(\int_0^\omega b(v)\,\mathrm{d}v)-1}\leqslant G(t,s)\leqslant\frac{\exp(\int_0^\omega b(v)\,\mathrm{d}v)}{\exp(\int_0^\omega b(v)\,\mathrm{d}v)-1}=:q\tag{2.7}$$

for all $s\in[t,t+\omega]$. Hence, for $x\in K$, we have

$$\|\Psi x\|\leqslant q\int_0^\omega |F(s,x(s-\tau_1(s)),\ldots,x(s-\tau_n(s)),(\Phi x)(s-\delta(s)))|\,\mathrm{d}s\tag{2.8}$$

and

$$\begin{aligned}
(\Psi x)(t)&\geqslant p\int_0^\omega |F(s,x(s-\tau_1(s)),\ldots,x(s-\tau_n(s)),(\Phi x)(s-\delta(s)))|\,\mathrm{d}s\\
&\geqslant\frac{p}{q}\|\Psi x\|\geqslant\sigma\|\Psi x\|.
\end{aligned}$$

Therefore, $(\Psi x)\in K$.

Claim 2. *Ψ is completely continuous.*

First we show that Ψ is continuous. According to our assumptions $F(t, z_1, z_2, \ldots, z_n, z_{n+1}) \in C(R^{n+2}, [0, \infty))$, $\tau_i(t)(i = 1, 2, \ldots, n)$, $\delta(t)$, $\sigma(t) \in C(R, R)$, $\eta(t)$, $a(t)$, $b(t) \in C(R, (0, \infty))$. We know that for any $\varepsilon > 0$, there exists a $\delta > 0$ such that for $\varphi, \psi \in C_\omega$,

$$\|\varphi - \psi\| < \delta, \quad 0 \leqslant s \leqslant \omega$$

imply

$$\begin{aligned} &\sup_{0\leqslant s\leqslant\omega} |F(s, \varphi(s - \tau_1(s)), \ldots, \varphi(s - \tau_n(s)), u(s - \delta(s))) \\ &\quad - F(s, \psi(s - \tau_1(s)), \ldots, \psi(s - \tau_n(s)), u(s - \delta(s)))| \\ &\quad\quad < \frac{\varepsilon}{q\omega}, \end{aligned}$$

where q is defined as (2.7). If $x, y \in K$ with $\|x - y\| < \delta$, also in view of (2.3), we have

$$\begin{aligned} &|(\Psi x)(t) - (\Psi y)(t)| \\ &\quad \leqslant \int_t^{t+\omega} G(t, s)|F(s, x(s - \tau_1(s)), \ldots, x(s - \tau_n(s)), (\Phi x)(s - \delta(s))) \\ &\qquad - F(s, x(s - \tau_1(s)), \ldots, x(s - \tau_n(s)), (\Phi x)(s - \delta(s)))|\, \mathrm{d}s \\ &\quad \leqslant q \int_0^{\omega} |F(s, x(s - \tau_1(s)), \ldots, x(s - \tau_n(s)), (\Phi x)(s - \delta(s))) \\ &\qquad - F(s, x(s - \tau_1(s)), \ldots, x(s - \tau_n(s)), (\Phi x)(s - \delta(s)))|\, \mathrm{d}s \\ &\quad < \varepsilon. \end{aligned}$$

This yields $\|(\Psi x) - (\Psi y)\| < \varepsilon$. Hence Ψ is continuous.

Secondly, we show that F is maps bounded sets into bounded sets. For convenience, we denote $BC(R, R^{n+1})$ as the sets of bounded continuous functions $\Phi\colon R \to R^{n+1}$. And define a vector $Ux(t) = (x(t - \tau_1(t)), \ldots, x(t - \tau_n(t)), (\Phi x)(t - \delta(t)))$, correspondingly, the norm of $Ux(t)$ is defined as $\|Ux(t)\| = \max\{\max_{i=1,\ldots,n}|x(t - \tau_i(t))|, \|(\Phi x)(t - \delta(t))\|\}$. In terms of our assumptions, we know that $Ux(s)$ is continuous in s, and there exists a $\delta > 0$ such that

$$Ux, Uy \in BC, \quad \|Ux\| \leqslant \lambda, \quad \|Uy\| \leqslant \lambda, \quad \|Ux - Uy\| < \delta$$

imply

$$|F(s, Ux(s)) - F(s, Uy(s))| < 1.$$

Choose a positive integer $N > \lambda/\delta$. Let $Ux \in BC$ and define $Ux^k(t) = Ux(t)k/N$ for $k = 0, 1, 2, \ldots, N$. If $\|Ux\| \leqslant \lambda$, then

$$\|Ux^k - Ux^{k-1}\| = \sup_{t\in R}\left|Ux(t)\frac{k}{N} - Ux(t)\frac{k-1}{N}\right| \leqslant \frac{\|Ux\|}{N} \leqslant \frac{\lambda}{N} < \delta.$$

Thus

$$|F(s, Ux^k) - F(s, Ux^{k-1})| < 1.$$

for all $s \in [0, \omega]$. This yields

$$\begin{aligned}|F(s, Ux(s))| &\leqslant \sum_{k=1}^{N} |F(s, Ux^k) - F(s, Ux^{k-1})| + |F(s, 0)| \\ &< N + \sup_{s\in[0,\omega]} |F(s, 0)| =: M_\lambda. \end{aligned} \tag{2.9}$$

Also in view of (2.8), we know for $s \in [0, \omega]$,

$$\|\Psi x\| \leqslant q \int_0^\omega |F(s, Ux(s))|\, \mathrm{d}s \leqslant q\omega M_\lambda.$$

Finally, for $t \in R$ we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(\Psi x)(t) &= G(t, t+\omega)F(t+\omega, Ux(t+\omega)) - G(t, t)F(t, Ux(t)) + b(t)(\Psi x)(t) \\ &= b(t)(\Psi x)(t) + (G(t, t+\omega) - G(t, t))F(t, Ux(t)) \\ &= b(t)(\Psi x)(t) + F(t, Ux(t)).\end{aligned}$$

This implies that

$$\begin{aligned}\left|\frac{\mathrm{d}}{\mathrm{d}t}(\Psi x)(t)\right| &\leqslant \|b(t)\| q \int_0^\omega |F(t, Ux(t))|\, \mathrm{d}t + |F(t, Ux(t))| \\ &\leqslant (q\omega\|b(t)\| + 1)M_\lambda,\end{aligned}$$

where

$$\|b(t)\| = \max_{t\in[0,\omega]} |b(t)|.$$

This shows that $\{(\Psi x) : x \in K, \|x\| \leqslant \lambda\}$ is a family of uniformly bounded and equicontinuous functions on $[0, \omega]$. By Ascoli–Arzela theorem, the function Ψ is completely continuous. It is obvious that if $x(t)$ is a fixed point of Ψ, then $x(t)$ is an ω-periodic solution of Eq. (1.8).

Now we are in a position to show that under the assumption (2.1) or (2.2), system (1.8) has at least one positive ω-periodic solution.

We only show that when (2.1) hold, the conclusion is true. The case when (2.2) hold could be proved by the same way. From the former expression of (2.1), we know that for any $\varepsilon > 0$, there exists a $\delta > 0$ such that $F(t, v)/|v| < \varepsilon$ if $|v| < \delta$. Chose $\varepsilon = 1/q\omega$ and define $R_1 = \delta/2$, then we have $|v| = R_1$ implies $F(t, v) < R_1/q\omega$, which yields

$$|(\Psi v)(t)| \leqslant q \int_t^{t+\omega} |F(s, v)|\, \mathrm{d}s \leqslant q\omega \frac{1}{q\omega} R_1 = R_1.$$

Denote $\Omega_1 = \{\psi \in C_\omega : \|\psi\| < R_1\}$. Then the above inequality implies that $\|(\Psi v)\| \leqslant |v|$ for $v \in K \cap \partial\Omega_1$. On the other hand, the latter expression of (2.1) implies that for any $M > 0$, there exists a $\Delta > 0$ such that $|F(t, v)|/|v| > M$ if $|v| > \Delta$. Chose $M = 1/p\omega$ and define $R_2 = 2\Delta$, then we have $|v| = R_2$ implies $|F(t, v)| > R_2/p\omega$, which yields

$$|(\Psi v)(t)| \geqslant p \int_t^{t+\omega} |F(s, v)|\, \mathrm{d}s \geqslant p\omega \frac{1}{p\omega} R_2 = R_2.$$

Denote $\Omega_2=\{\psi\in C_\omega:\|\psi\|<R_2\}$. Then the above inequality implies that $\|(\Psi v)\|\leqslant|v|$ for $v\in K\cap\partial\Omega_2$. By Lemma 2.1, Ψ has a fixed point in $K\cap(\bar{\Omega}_2\backslash\Omega_1)$. Hence, system (1.8) has a positive ω-periodic solution. This completes the proof. □

Theorem 2.2. *Assume that* (2.1) *or* (2.2) *hold. Then system* (1.9) *has at least one positive ω-periodic solution.*

The proof is similar to that of Theorem 2.1, we omit it here.

Now we reach a position to derive the conditions which guarantee system (1.8) has a unique positive ω-periodic solution that attracts all positive solutions of (1.8). For convenience, for the rest of this section, we suppose that $\tau_1(t)=\tau_2(t)=\cdots=\tau_n(t)=\sigma(t)=\tau$ and $\delta(t)=0$, here τ is a positive constant.

First we show that the following proposition hold.

Proposition. *If the periodic solution $x^*(t)$ of* (2.4) *attracts all the positive solutions of it, then the periodic solution $(x^*(t),u^*(t))$ of system* (1.8) *attracts all the positive solutions of system* (1.8).

Proof. Let $(x(t),u(t))$ be an arbitrary positive solution of (1.8), then $x(t)$ is a positive solution of (2.4). If $x^*(t)$ attracts all the positive solutions of (2.4), that is,

$$\lim_{t\to+\infty}(x(t)-x^*(t))=0. \tag{2.10}$$

We only need to prove that

$$\lim_{t\to+\infty}(u(t)-u^*(t))=0. \tag{2.11}$$

In view of the second equation of (1.8), we can obtain

$$u(t)=u(0)\exp\left\{-\int_0^t\eta(s)\,\mathrm{d}s\right\}+\exp\left\{-\int_0^t\eta(s)\,\mathrm{d}s\right\}\int_0^t\exp\left\{\int_0^t\eta(s)\,\mathrm{d}s\right\}a(t)x(t-\sigma(t))\,\mathrm{d}t$$

and

$$\begin{aligned}u^*(t)=&\,u^*(0)\exp\left\{-\int_0^t\eta(s)\,\mathrm{d}s\right\}\\&+\exp\left\{-\int_0^t\eta(s)\,\mathrm{d}s\right\}\int_0^t\exp\left\{\int_0^t\eta(s)\,\mathrm{d}s\right\}a(t)x^*(t-\sigma(t))\,\mathrm{d}t.\end{aligned}$$

From the above two equalities, one can see that

$$\begin{aligned}u(t)-u^*(t)=&\exp\left\{-\int_0^t\eta(s)\,\mathrm{d}s\right\}[u(0)-u^*(0)]+\exp\left\{-\int_0^t\eta(s)\,\mathrm{d}s\right\}\\&\times\int_0^t\exp\left\{\int_0^t\eta(s)\,\mathrm{d}s\right\}a(t)[x(t-\sigma(t))-x^*(t-\sigma(t))]\,\mathrm{d}t.\end{aligned}$$

In terms of (2.10), it is obvious that

$$\lim_{t\to+\infty}(x(t-\sigma(t))-x^*(t-\sigma(t)))=0.$$

Also notice that $\eta(t) \in C(R, (0, \infty))$ and $\eta(t)$ is ω-periodic, hence

$$\lim_{t \to +\infty} \exp\left\{-\int_0^t \eta(s)\,\mathrm{d}s\right\} = 0.$$

Therefore (2.11) hold. The proof is complete. □

Let $(x^*(t), u^*(t))$ be a positive ω-periodic solution of (1.8). Set

$$z(t) = x(t) - x^*(t) \tag{2.12}$$

and denote

$$F(t, x(t-\tau), \ldots, x(t-\tau), (\Phi x)(t)) = F(t, x(t-\tau), (\Phi x)(t)),$$

then

$$\begin{aligned} z'(t) &= x'(t) - x^{*\prime}(t) \\ &= -b(t)z(t) + F(t, x^*(t-\tau) + z(t-\tau), (\Phi x)(t)) - F(t, x^*(t-\tau), (\Phi x^*)(t)) \\ &= -b(t)z(t) + \frac{\partial F(t, \xi(t), \zeta(t))}{\partial u_1} z(t-\tau) + \frac{\partial F(t, \xi(t), \zeta(t))}{\partial u_2}\frac{\partial u_2(\varsigma(t))}{\partial x(t-\tau)} z(t-\tau), \end{aligned} \tag{2.13}$$

where $\xi(t)$, $\varsigma(t)$ lies between $x^*(t-\tau)$ and $x(t-\tau)$; $\zeta(t)$ lies between $(\Phi x)^*(t)$ and $(\Phi x)(t)$. And $\partial u_2(\varsigma(t))/\partial x(t-\tau)$ can be computed as follows.

Notice that $u(t) = (\Phi x)(t)$ satisfying

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\eta(t)u(t) + a(t)x(t-\tau),$$

which implies

$$\frac{\mathrm{d}\left(\frac{\partial u}{\partial x(t-\tau)}\right)}{\mathrm{d}t} = -\eta(t)\frac{\partial u}{\partial x(t-\tau)} + a(t). \tag{2.14}$$

Let

$$\frac{\partial u}{\partial x(t-\tau)} = y,$$

then (2.14) becomes

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\eta(t)y + a(t).$$

We may solve y from the above ODE that

$$y(t) = \exp\left\{-\int_0^t \eta(t)\,\mathrm{d}t\right\}\left(y(0) + \int_0^t a(t)\,\exp\left\{\int_0^t \eta(t)\,\mathrm{d}t\right\}\mathrm{d}t\right).$$

This means

$$\frac{\partial u}{\partial x(t-\tau)} = \exp\left\{-\int_0^t \eta(t)\,\mathrm{d}t\right\}\left(y(0) + \int_0^t a(t)\exp\left\{\int_0^t \eta(t)\,\mathrm{d}t\right\}\mathrm{d}t\right).$$

Denote

$$r(t) = -\left[\frac{\partial F(t, \xi(t), \zeta(t))}{\partial u_1} + \frac{\partial F(t, \xi(t), \zeta(t))}{\partial u_2} \cdot \frac{\partial u_2(\varsigma(t))}{\partial x(t-\tau)}\right],$$

make change of variables

$$z(t) = \exp\left\{-\int_0^t b(s)\,\mathrm{d}s\right\} w(t), \tag{2.15}$$

then (2.13) becomes

$$w'(t) + r(t)\exp\left\{\int_{t-\tau}^t b(s)\,\mathrm{d}s\right\} w(t-\tau) = 0. \tag{2.16}$$

Then by Theorem 2.1.5 in [16], we conclude the following result.

Theorem 2.3. *Assume that* (2.1) *or* (2.2) *hold. Furthermore suppose that* $\mathsf{F} \in C^1(R^3, [0, \infty))$,

$$r(t) > 0, \quad \textit{for sufficiently large } t,$$
$$\lim_{t\to+\infty}\sup \int_{t-\tau}^t r(t)\exp\left\{\int_{t-\tau}^t b(s)\,\mathrm{d}s\right\}\mathrm{d}t < \frac{\pi}{2} \tag{2.17}$$

and there exists a $T > 0$ *such that*

$$\int_T^{+\infty} r(t)\exp\left\{\int_{t-\tau}^t b(s)\,\mathrm{d}s\right\}\mathrm{d}t = +\infty, \tag{2.18}$$

where F *and* $r(t)$ *are defined as above. Then every positive solution* $(x(t), u(t))$ *of system* (1.8) *tends to* $(x^*(t), u^*(t))$.

Similarly, for system (1.9), we have

Theorem 2.4. *Assume that* (2.1) *or* (2.2) *hold. Furthermore suppose that* $\mathsf{F} \in C^1(R^{n+2}, [0, \infty))$,

$$r(t) < 0, \quad \textit{for sufficiently large } t,$$
$$\lim_{t\to+\infty}\sup \int_{t-\tau}^t -r(t)\ \exp\left\{-\int_{t-\tau}^t b(s)\,\mathrm{d}s\right\}\mathrm{d}t < \frac{\pi}{2} \tag{2.19}$$

and there exists a $T > 0$ *such that*

$$\int_T^{+\infty} -r(t)\exp\left\{-\int_{t-\tau}^t b(s)\,\mathrm{d}s\right\}\mathrm{d}t = +\infty,$$

where F *and* $r(t)$ *are defined as above. Then every positive solution* $(x(t), u(t))$ *of system* (1.9) *tends to* $(x^*(t), u^*(t))$.

Theorem 2.5. *Assume that* (2.17), (2.18) *and* (2.1) *or* (2.2) *hold. Let* $x(t)$ *be a positive solution of system* (2.4) *which does not oscillate about* $x^*(t)$, *then*

$$\lim_{t\to+\infty}(x(t) - x^*(t)) = 0.$$

Proof. Since system (1.8) is equivalent to system (2.16), assume that $x(t) > x^*(t)$ for sufficiently large t (the proof when $x(t) < x^*(t)$ is similar and hence to be omitted). Notice that the transformations (2.12) and (2.15), we have $w(t) > 0,$ for t sufficiently large. Thus

$$w'(t) = -r(t)\exp\left\{\int_{t-\tau}^{t} b(s)\,\mathrm{d}s\right\} w(t-\tau) < 0. \tag{2.20}$$

This implies that $w(t)$ is decreasing, and therefore

$$\lim_{t\to+\infty} w(t) = \alpha \in [0, +\infty).$$

Now we only need to show that $\alpha = 0$. If $\alpha > 0$, then there exist $\varepsilon > 0$ and $T > 0$ such that for $t \geqslant T$, $0 < \alpha - \varepsilon < w(t) < \alpha + \varepsilon$. Then from (2.20), we have

$$w'(t) = -r(t)\exp\left\{\int_{t-\tau}^{t} b(s)\,\mathrm{d}s\right\} w(t-\tau) > -r(t)\exp\left\{\int_{t-\tau}^{t} b(s)\,\mathrm{d}s\right\}(\alpha+\varepsilon),$$

integrate both sides of the above inequality from T to $+\infty$, we obtain

$$\alpha - w(T) > \int_{T}^{+\infty} -r(t)\exp\left\{\int_{t-\tau}^{t} b(s)\,\mathrm{d}s\right\}(\alpha+\varepsilon)\,\mathrm{d}t.$$

This contradicts the assumption (2.18). Hence $\alpha = 0$ and the proof is complete. □

3. Applications

In this section, we denote

$$f^{\mathrm{U}} = \max_{t\in[0,\omega]} f(t), \qquad f^{\mathrm{L}} = \max_{t\in[0,\omega]} f(t),$$

where $f(t)$ is a ω-periodic continuous function. Now we apply the main result obtained in the previous section to some well known models in population dynamics.

Example 3.1. For the periodic delay Hematopoiesis model [23,29,31],

$$\begin{aligned} \frac{\mathrm{d}x(t)}{\mathrm{d}t} &= -b(t)x(t) + \frac{\beta(t)}{1+[x(t-\tau(t))]^m} + c(t)u(t-\delta(t)),\\ \frac{\mathrm{d}u}{\mathrm{d}t} &= -\eta(t)u(t) + a(t)x(t-\sigma(t)), \end{aligned} \tag{3.1}$$

where $m > 0$ is a constant, $b(t), \beta(t), \eta(t), a(t), c(t), \tau(t), \delta(t), \sigma(t) \in C(R, (0, +\infty))$ and they are all ω-periodic. We can obtain the existence of positive periodic solution and the sufficient conditions for the global attractivity being incorporated in the following two theorems.

Theorem 3.1. *System* (3.1) *has at least one positive ω-periodic solution.*

Proof. For any positive solution of (3.1), Eq. (3.1) is equivalent to the following equation

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -b(t)x(t) + \frac{\beta(t)}{1+[x(t-\tau(t))]^m} + c(t)\int_{t-\delta(t)}^{t-\delta(t)+\omega} g(t-\delta(t), s)a(s)x(s-\sigma(s))\,\mathrm{d}s.$$

Denote

$$F(x(t-\tau(t)), x(t-\delta(t))) = \frac{\beta(t)}{1+[x(t-\tau(t))]^m} + c(t)\int_{t-\delta(t)}^{t-\delta(t)+\omega} g(t-\delta(t), s)a(s)x(s-\sigma(s))\,\mathrm{d}s.$$

Clearly, $F(x(t-\tau(t)), x(t-\delta(t))) \in C(R^2, [0, +\infty))$, and denote

$$x_M = \max\{x(t-\tau(t)), x(t-\delta(t))\}.$$

Then we have

$$\lim_{x_M\to 0} \frac{F(x(t-\tau(t)), x(t-\delta(t)))}{x_M} = +\infty, \qquad \lim_{x_M\to +\infty} \frac{F(x(t-\tau(t)), x(t-\delta(t)))}{x_M} = 0.$$

By Theorem 2.1, we can conclude that system (3.1) has at least one positive ω-periodic solution.

In what follows, we will give sufficient conditions for the globally asymptotically stability of system (3.1). First, we introduce a lemma which is useful to establish the global stability result. □

Lemma 3.1. *Assume that $f(t)$, $g(t)$ are continuous nonnegative functions defined on the interval $[\alpha, \beta]$ satisfying*

$$f(t) \leqslant K + \int_\alpha^t f(s-\tau)g(s)\,\mathrm{d}s, \quad \text{for } \alpha+\tau \leqslant t \leqslant \beta,$$

where $K \geqslant 0$, $0 \leqslant \tau \leqslant \beta-\alpha$ are constants. Then

$$f(t) \leqslant K \exp\left\{\int_\alpha^t g(s)\,\mathrm{d}s\right\}, \quad \text{for } \alpha+\tau \leqslant t \leqslant \beta.$$

Theorem 3.2. *Suppose that*

$$\tau(t) = \sigma(t) = n\omega, \quad \delta(t) = 0, \tag{3.2}$$

where n is a positive integer, furthermore assume

$$\begin{aligned} mB_1\beta(t) > {} & c(t)\exp\left\{-\int_0^t \eta(t)\,\mathrm{d}t\right\}\int_0^t a(t)\exp\left\{\int_0^t \eta(t)\,\mathrm{d}t\right\}\mathrm{d}t \\ & \times \limsup_{t\to\infty} \int_{t-\tau}^t \left(mB_2\beta(t) - c(t)\exp\left\{-\int_0^t \eta(t)\,\mathrm{d}t\right\}\int_0^t a(t)\exp\left\{\int_0^t \eta(t)\,\mathrm{d}t\right\}\mathrm{d}t\right) \\ & \times \exp\left\{\int_{t-\tau}^t b(s)\,\mathrm{d}s\right\}\mathrm{d}t \\ < {} & \frac{\pi}{2} \end{aligned}$$

and there exists a $T>0$ *such that*

$$\int_T^{+\infty}\left(mB_1\beta(t)-c(t)\exp\left\{-\int_0^t\eta(t)\,\mathrm{d}t\right\}\int_0^t a(t)\exp\left\{\int_0^t\eta(t)\,\mathrm{d}t\right\}\mathrm{d}t\right)\\ \times\exp\left\{\int_{t-\tau}^t b(s)\,\mathrm{d}s\right\}\mathrm{d}t=+\infty,$$

where B_1, B_2 *are defined in the following. Then every positive solution* $(x(t),u(t))$ *of system* (3.1) *tends to* $(x^*(t),u^*(t))$.

Proof. From (2.4), we know

$$\begin{aligned}\frac{\mathrm{d}x(t)}{\mathrm{d}t}&=-b(t)x(t)+\frac{\beta(t)}{1+[x(t-n\omega)]^m}+c(t)\int_t^{t+\omega}g(t,s)a(s)x(s-n\omega)\,\mathrm{d}s\\&\geqslant -b(t)x(t)+\frac{\beta(t)}{1+[x(t-n\omega)]^m}.\end{aligned}$$

Make auxiliary equation

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t}=-b(t)p(t)+\frac{\beta(t)}{1+[p(t-n\omega)]^m}.$$

By Theorem 2.7 in [29], we know that there exists a T such that for all $t\geqslant T$,

$$p(t)>p_1\exp(-b^{\mathrm{U}}m\omega):=A_1,$$

where p_1 is the unique positive zero of the function

$$f_1(p)=\frac{\beta^{\mathrm{L}}}{1+p^m}-b^{\mathrm{U}}p.$$

Then we have

$$x(t)>A_1.$$

On the other hand, consider the equation

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}=-b(t)x(t)+\frac{\beta(t)}{1+[x(t-n\omega)]^m}+c(t)\int_t^{t+\omega}g(t,s)a(s)x(s-n\omega)\,\mathrm{d}s. \tag{3.3}$$

Now we prove that $x(t)$ is bounded from above in the interval $[0,+\infty)$. It is easy to know that if $x'(t)<0$ for sufficiently large t, this proposition hold true [1]. If $x'(t)>0$ for sufficiently large t, we have

$$b(t)x(t)\leqslant\beta(t)+c(t)\int_t^{t+\omega}g(t,s)a(s)x(s-n\omega)\,\mathrm{d}s,$$

which implies

$$\begin{aligned}
x(t) &\leqslant \frac{\beta(t)}{b(t)} + \frac{c(t)}{b(t)} \int_t^{t+\omega} g(t,s)a(s)x(s-n\omega)\,\mathrm{d}s \\
&\leqslant \left(\frac{\beta(t)}{b(t)}\right)^{\mathrm{U}} + \left(\frac{c(t)}{b(t)}\right)^{\mathrm{U}} g(0,\omega) \int_t^{t+\omega} a(s)x(s-n\omega)\,\mathrm{d}s \\
&= \left(\frac{\beta(t)}{b(t)}\right)^{\mathrm{U}} + \left(\frac{c(t)}{b(t)}\right)^{\mathrm{U}} g(0,\omega) \int_{t-\omega}^{t} a(s)x(s-(n-1)\omega)\,\mathrm{d}s \\
&\leqslant \left(\frac{\beta(t)}{b(t)}\right)^{\mathrm{U}} + \left(\frac{c(t)}{b(t)}\right)^{\mathrm{U}} g(0,\omega) \int_0^{t} a(s)x(s-(n-1)\omega)\,\mathrm{d}s.
\end{aligned} \tag{3.4}$$

In terms of Lemma 3.1, we obtain

$$x(t) \leqslant \left(\frac{\beta(t)}{b(t)}\right)^{\mathrm{U}} \exp\left\{\left(\frac{c(t)}{b(t)}\right)^{\mathrm{U}} g(0,\omega) \int_0^t a(s)\,\mathrm{d}s\right\}, \quad \text{for } t \geqslant (n-1)\omega,$$

using the second inequility of (3.4), we have

$$\begin{aligned}
x(t) &\leqslant \left(\frac{\beta(t)}{b(t)}\right)^{\mathrm{U}} + \left(\frac{c(t)}{b(t)}\right)^{\mathrm{U}} g(0,\omega) \\
&\quad \times \int_0^{\omega} \left[a(s)\left(\frac{\beta(t)}{b(t)}\right)^{\mathrm{U}} \exp\left\{\left(\frac{c(t)}{b(t)}\right)^{\mathrm{U}} g(0,\omega) \int_0^s a(s)\,\mathrm{d}s\right\}\right] \mathrm{d}s \\
&:= A_2.
\end{aligned}$$

Either if $x'(t)$ is oscillate, denote its zeros as sequence $\{t_n\}_{n=1,2,\ldots,}$ without loss of generality, suppose that $x'(t) \leqslant 0$ on $[t_{k-1}, t_k]$, $x'(t) \geqslant 0$ on $[t_k, t_{k+1}]$ and $x'(t) \leqslant 0$ on $[t_{k+1}, t_{k+2}]$, where $k > 1$ is an integer. Then

$$x(t) \leqslant x(t_{k-1}), \quad t \in [t_{k-1}, t_k];$$

$$x(t) \leqslant x(t_{k+1}), \quad t \in [t_k, t_{k+1}];$$

$$x(t_{k+1}) \geqslant x(t), \quad t \in [t_{k+1}, t_{k+2}].$$

By the same discussion as above, we have

$$x(t_{k+1}) \leqslant A_2,$$

then by the method of steps, we know for sufficiently large t,

$$x(t) \leqslant A_2.$$

Since

$$
\begin{aligned}
r(t) &= -\left[\frac{\partial F(t,\xi(t),\zeta(t))}{\partial u_1} + \frac{\partial F(t,\xi(t),\zeta(t))}{\partial u_2}\frac{\partial u_2(\varsigma(t))}{\partial x(t-n\omega)}\right] \\
&= \frac{m\beta(t)\xi^{m-1}(t)}{(1+\xi^m(t))^2} - c(t)\exp\left\{-\int_0^t \eta(t)\,\mathrm{d}t\right\}\left(y(0)+\int_0^t a(t)\exp\left\{\int_0^t \eta(t)\,\mathrm{d}t\right\}\mathrm{d}t\right) \\
&> 0,
\end{aligned}
$$

denote

$$
\min_{\xi(t)\in[A_1,A_2]} \frac{\xi^{m-1}(t)}{(1+\xi^m(t))^2} = B_1, \qquad \max_{\xi(t)\in[A_1,A_2]} \frac{\xi^{m-1}(t)}{(1+\xi^m(t))^2} = B_2.
$$

Then by Theorem 2.5, we complete the proof. □

We remark that when our main results are applied to the models (1.2)–(1.4), the conditions for the existence of positive periodic solutions are the same as the previous results [12,21]. However, when our results on the global attractivity are applied to these models, the conditions are more easily verified. Moreover, for such types models such as (1.5)–(1.7), the previous articles cannot solve its existence of positive periodic solutions as well as the global attractivity of the positive solutions. In our present paper, we solve such kind of problems, though our conditions for the global attractivity looks rather complicated.

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